



# *A Story of Ratios:* A Curriculum Overview for Grades 6–8

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## Introduction

This document provides an overview of the academic year for Grades 6 through 8, beginning with a curriculum map and followed by detailed grade-level descriptions.

The curriculum map is a chart that shows, at a glance, the sequence of modules comprising each grade of the Grades 6 through 8 curricula. The map also indicates the approximate number of instructional days designated for each module of each grade. Details that elaborate on the curriculum map are found in the grade-level descriptions.

Each grade-level description begins with a list of the six to seven modules that comprise the instruction of that grade. That introductory component is followed by three sections: the Summary of Year, the Rationale for Module Sequence, and the alignment chart with the grade-level standards.

The Summary of Year portion of each grade level includes four pieces of information:

- The critical instructional areas for the grade, as described in the Common Core State Standards for Mathematics<sup>1</sup> (CCSS-M)
- The Key Areas of Focus<sup>2</sup> for the grade
- The Required Fluencies for the grade
- The Major Emphasis Clusters<sup>3</sup> for the grade

The Rationale for Module Sequence portion of each grade level provides a brief description of the instructional focus of each module for that grade and explains the developmental sequence of the mathematics.

The alignment chart for each grade lists the standards that are addressed in each module of the grade. Note that when a cluster is referred to without a footnote, it is taught in its entirety. There are also times when footnotes are relevant to particular standards within a cluster. All standards for each grade have been carefully included in the module sequence. Some standards are deliberately included in more than one module so that a strong foundation can be built over time.

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<sup>1</sup> [http://www.corestandards.org/wp-content/uploads/Math\\_Standards1.pdf](http://www.corestandards.org/wp-content/uploads/Math_Standards1.pdf)

<sup>2</sup> [http://www.achievethecore.org/downloads/E0702\\_Description\\_of\\_the\\_Common\\_Core\\_Shifts.pdf](http://www.achievethecore.org/downloads/E0702_Description_of_the_Common_Core_Shifts.pdf)

<sup>3</sup> <http://www.parcconline.org/resources/educator-resources/model-content-frameworks/mathematics-model-content-framework>

	Grade 6	Grade 7	Grade 8		2015-16*
1st TRIMESTER	M1: Ratios and Unit Rates (35 days)	M1: Ratios and Proportional Relationships (30 days)	M1: Integer Exponents and Scientific Notation (20 days)	1st QUARTER	
	M2: Arithmetic Operations Including Division of Fractions (25 days)	M2: Rational Numbers (30 days)	M2: The Concept of Congruence (25 days)		
2nd TRIMESTER	M3: Rational Numbers (25 days)	M3: Expressions and Equations (35 days)	M3: Similarity (25 days)	2nd QUARTER	
	M4: Expressions and Equations (45 days)	M4: Percent and Proportional Relationships (25 days)	M4: Linear Equations (40 days)		
3rd TRIMESTER	M5: Area, Surface Area, and Volume Problems (25 days)	M5: Statistics and Probability (25 days)	M5: Examples of Functions from Geometry (15 days)	3rd QUARTER	
	M6: Statistics (25 days)	M6: Geometry (35 days)	M6: Linear Functions (20 days)		
			M7: Introduction to Irrational Numbers Using Geometry (35 days)	4th QUARTER	

Key:					
Number	Geometry	Ratios and Proportions	Expressions and Equations	Statistics and Probability	Functions

\*The columns indicating trimesters and quarters are provided to give you a rough guideline. Please use this additional column for your own pacing considerations based on the specific dates of your academic calendar.

## Sequence of Grade 6 Modules Aligned with the Standards

Module 1: Ratios and Unit Rates

Module 2: Arithmetic Operations Including Division of Fractions

Module 3: Rational Numbers

Module 4: Expressions and Equations

Module 5: Area, Surface Area, and Volume Problems

Module 6: Statistics

### Summary of Year

Grade 6 mathematics is about (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

**Key Areas of Focus for Grade 6:** Ratios and proportional reasoning; early expressions and equations

**Required Fluency:** 6.NS.B.2 Multi-digit division  
6.NS.B.3 Multi-digit decimal operations

Major Emphasis Clusters
Ratios and Proportional Relationships <ul style="list-style-type: none"> <li>Understand ratio concepts and use ratio reasoning to solve problems.</li> </ul>
The Number System <ul style="list-style-type: none"> <li>Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</li> <li>Apply and extend previous understandings of numbers to the system of rational numbers.</li> </ul>
Expressions and Equations <ul style="list-style-type: none"> <li>Apply and extend previous understandings of arithmetic to algebraic expressions.</li> <li>Reason about and solve one-variable equations and inequalities.</li> <li>Represent and analyze quantitative relationships between dependent and independent variables.</li> </ul>

### Rationale for Module Sequence in Grade 6

In Module 1, students build on their prior work in measurement and in multiplication and division as they study the concepts and language of ratios and unit rates. They use proportional reasoning to solve problems. In particular, students solve ratio and rate problems using tape diagrams, tables of equivalent ratios, double number line diagrams, and equations. They plot pairs of values generated from a ratio or rate on the first quadrant of the coordinate plane.

Students expand their understanding of the number system and build their fluency in arithmetic operations in Module 2. Students learned in Grade 5 to divide whole numbers by unit fractions and unit fractions by whole numbers. Now, they apply and extend their understanding of multiplication and division to divide fractions by fractions. The meaning of this operation is connected to real-world problems as students are asked to create and solve fraction division word problems. Students continue (from Grade 5) to build fluency with adding, subtracting, multiplying, and dividing multi-digit decimal numbers using the standard algorithms.

Major themes of Module 3 are to understand rational numbers as points on the number line and to extend previous understandings of numbers to the system of rational numbers, which now include negative numbers. Students extend coordinate axes to represent points in the plane with negative number coordinates and, as part of doing so, see that negative numbers can represent quantities in real-world contexts. They use the number line to order numbers and to understand the absolute value of a number. They begin to solve real-world and mathematical problems by graphing points in all four quadrants, a concept that continues throughout to be used into high school and beyond.

With their sense of number expanded to include negative numbers, in Module 4 students begin formal study of algebraic expressions and equations. Students learn equivalent expressions by continuously relating algebraic expressions back to arithmetic and the properties of arithmetic (commutative, associative, and distributive). They write, interpret, and use expressions and equations as they reason about and solve one-variable equations and inequalities and analyze quantitative relationships between two variables.

Module 5 is an opportunity to practice the material learned in Module 4 in the context of geometry; students apply their newly acquired capabilities with expressions and equations to solve for unknowns in area, surface area, and volume problems. They find the area of triangles and other two-dimensional figures and use the formulas to find the volumes of right rectangular prisms with fractional edge lengths. Students use negative numbers in coordinates as they draw lines and polygons in the coordinate plane. They also find the lengths of sides of figures, joining points with the same first coordinate or the same second coordinate, and apply these techniques to solve real-world and mathematical problems.

In Module 6, students develop an understanding of statistical variability and apply that understanding as they summarize, describe, and display distributions. In particular, careful attention is given to measures of center and variability.

Alignment Chart<sup>4</sup>

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 6 Modules
<p><b>Module 1:</b> <b>Ratios and Unit Rates</b> (35 days)</p>	<p><b>Understand ratio concepts and use ratio reasoning to solve problems.</b></p> <p><b>6.RP.A.1</b> Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. <i>For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”</i></p> <p><b>6.RP.A.2</b> Understand the concept of a unit rate <math>a/b</math> associated with a ratio <math>a:b</math> with <math>b \neq 0</math>, and use rate language in the context of a ratio relationship. <i>For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is <math>3/4</math> cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”<sup>5</sup></i></p> <p><b>6.RP.A.3</b> Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</p> <ol style="list-style-type: none"> <li>Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</li> <li>Solve unit rate problems including those involving unit pricing and constant speed. <i>For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?</i></li> <li>Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means <math>30/100</math> times the quantity); solve problems involving finding the whole, given a part and the percent.</li> <li>Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.</li> </ol>

<sup>4</sup> When a cluster is referred to in this chart without a footnote, the cluster is taught in its entirety.

<sup>5</sup> Expectations for unit rates in this grade are limited to non-complex fractions.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 6 Modules
<p><b>Module 2:</b> <b>Arithmetic Operations Including Division of Fractions</b> (25 days)</p>	<p><b>Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</b></p> <p><b>6.NS.A.1</b> Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. <i>For example, create a story context for <math>(2/3) \div (3/4)</math> and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that <math>(2/3) \div (3/4) = 8/9</math> because <math>3/4</math> of <math>8/9</math> is <math>2/3</math>. (In general, <math>(a/b) \div (c/d) = ad/bc</math>.) How much chocolate will each person get if 3 people share <math>1/2</math> lb of chocolate equally? How many <math>3/4</math>-cup servings are in <math>2/3</math> of a cup of yogurt? How wide is a rectangular strip of land with length <math>3/4</math> mi and area <math>1/2</math> square mi?</i></p> <p><b>Compute fluently with multi-digit numbers and find common factors and multiples.</b></p> <p><b>6.NS.B.2</b> Fluently divide multi-digit numbers using the standard algorithm.<sup>6</sup></p> <p><b>6.NS.B.3</b> Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.<sup>7</sup></p> <p><b>6.NS.B.4</b> Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. <i>For example, express <math>36 + 8</math> as <math>4(9 + 2)</math>.</i></p>
<p><b>Module 3:</b> <b>Rational Numbers</b> (25 days)</p>	<p><b>Apply and extend previous understandings of numbers to the system of rational numbers.</b></p> <p><b>6.NS.C.5</b> Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.</p> <p><b>6.NS.C.6</b> Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane</p>

<sup>6</sup> This fluency standard begins in this module and is practiced throughout the remainder of the year.

<sup>7</sup> This fluency standard begins in this module and is practiced throughout the remainder of the year.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 6 Modules
	<p>with negative number coordinates.</p> <ol style="list-style-type: none"> <li>Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., <math>-(-3) = 3</math>, and that 0 is its own opposite.</li> <li>Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.</li> <li>Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.</li> </ol> <p><b>6.NS.C.7</b> Understand ordering and absolute value of rational numbers.</p> <ol style="list-style-type: none"> <li>Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. <i>For example, interpret <math>-3 &gt; -7</math> as a statement that <math>-3</math> is located to the right of <math>-7</math> on a number line oriented from left to right.</i></li> <li>Write, interpret, and explain statements of order for rational numbers in real-world contexts. <i>For example, write <math>-3^{\circ}\text{C} &gt; -7^{\circ}\text{C}</math> to express the fact that <math>-3^{\circ}\text{C}</math> is warmer than <math>-7^{\circ}\text{C}</math>.</i></li> <li>Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. <i>For example, for an account balance of <math>-30</math> dollars, write <math> -30  = 30</math> to describe the size of the debt in dollars.</i></li> <li>Distinguish comparisons of absolute value from statements about order. <i>For example, recognize that an account balance less than <math>-30</math> dollars represents a debt greater than 30 dollars.</i></li> </ol> <p><b>6.NS.C.8</b> Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.</p>



Module and Approximate Number of Instructional Days	Standards Addressed in Grade 6 Modules
<p><b>Module 4:</b> <b>Expressions and Equations</b> (45 days)</p>	<p><b>Apply and extend previous understandings of arithmetic to algebraic expressions.<sup>8</sup></b></p> <p><b>6.EE.A.1</b> Write and evaluate numerical expressions involving whole-number exponents.</p> <p><b>6.EE.A.2</b> Write, read, and evaluate expressions in which letters stand for numbers.</p> <ol style="list-style-type: none"> <li>Write expressions that record operations with numbers and with letters standing for numbers. <i>For example, express the calculation “Subtract <math>y</math> from 5” as <math>5 - y</math>.</i></li> <li>Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. <i>For example, describe the expression <math>2(8 + 7)</math> as a product of two factors; view <math>(8 + 7)</math> as both a single entity and a sum of two terms.</i></li> <li>Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). <i>For example, use the formulas <math>V = s^3</math> and <math>A = 6s^2</math> to find the volume and surface area of a cube with sides of length <math>s = 1/2</math>.</i></li> </ol> <p><b>6.EE.A.3</b> Apply the properties of operations to generate equivalent expressions. <i>For example, apply the distributive property to the expression <math>3(2 + x)</math> to produce the equivalent expression <math>6 + 3x</math>; apply the distributive property to the expression <math>24x + 18y</math> to produce the equivalent expression <math>6(4x + 3y)</math>; apply properties of operations to <math>y + y + y</math> to produce the equivalent expression <math>3y</math>.</i></p> <p><b>6.EE.A.4</b> Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). <i>For example, the expressions <math>y + y + y</math> and <math>3y</math> are equivalent because they name the same number regardless of which number <math>y</math> stands for.</i></p>

<sup>8</sup> 6.EE.A.2c is also taught in Module 4 in the context of geometry.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 6 Modules
	<p><b>Reason about and solve one-variable equations and inequalities.<sup>9</sup></b></p> <p><b>6.EE.B.5</b> Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.</p> <p><b>6.EE.B.6</b> Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</p> <p><b>6.EE.B.7</b> Solve real-world and mathematical problems by writing and solving equations of the form <math>x + p = q</math> and <math>px = q</math> for cases in which <math>p</math>, <math>q</math>, and <math>x</math> are all nonnegative rational numbers.</p> <p><b>6.EE.B.8</b> Write an inequality of the form <math>x &gt; c</math> or <math>x &lt; c</math> to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form <math>x &gt; c</math> or <math>x &lt; c</math> have infinitely many solutions; represent solutions of such inequalities on number line diagrams.</p> <p><b>Represent and analyze quantitative relationships between dependent and independent variables.</b></p> <p><b>6.EE.C.9</b> Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. <i>For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation <math>d = 65t</math> to represent the relationship between distance and time.</i></p>
<p><b>Module 5:</b> <b>Area, Surface Area, and Volume Problems</b> (25 days)</p>	<p><b>Solve real-world and mathematical problems involving area, surface area, and volume.</b></p> <p><b>6.G.A.1</b> Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.</p>

<sup>9</sup> Except for 6.EE.B.8, this cluster is also taught in Module 4 in the context of geometry.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 6 Modules
	<p><b>6.G.A.2</b> Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas <math>V = l w h</math> and <math>V = b h</math> to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.</p> <p><b>6.G.A.3</b> Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.</p> <p><b>6.G.A.4</b> Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.</p>
<p><b>Module 6:</b> <b>Statistics</b> (25 days)</p>	<p><b>Develop understanding of statistical variability.</b></p> <p><b>6.SP.A.1</b> Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. <i>For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.</i></p> <p><b>6.SP.A.2</b> Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.</p> <p><b>6.SP.A.3</b> Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.</p> <p><b>Summarize and describe distributions.</b></p> <p><b>6.SP.B.4</b> Display numerical data in plots on a number line, including dot plots, histograms, and box plots.</p> <p><b>6.SP.B.5</b> Summarize numerical data sets in relation to their context, such as by:</p> <ol style="list-style-type: none"> <li>Reporting the number of observations.</li> </ol>

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 6 Modules
	<ul style="list-style-type: none"><li>b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.</li><li>c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.</li><li>d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.</li></ul>